Supplemental WS #9

① Degree: 3

Lead Coefficient: 1

$f(x) = x^3 + 3x^2 - 9x + 5$ $\quad \pm 1, \pm 5$

$P(1) = 1 + 3 - 9 + 5 = 0$ $\quad$ so $x - 1$ is a factor

1	1	3	-9	5
		1	4	-5
	1	4	-5	0

$f(x) = (x-1)(x^2 + 4x - 5)$

$f(x) = (x-1)(x-1)(x+5)$

Zeros: 1 w/ multiplicity 2

-5 w/ multiplicity 1

y-int: 5

② Degree: 3

Lead Coefficient: 2

$f(x) = 2x^3 - 9x^2 + 7x + 6$ $\quad \pm 1, \pm 2, \pm 3, \pm 6$

$P(1) = 2 - 9 + 7 + 6 = 6$

$P(-1) = -2 - 9 - 7 + 6 = -12$

$P(2) = 2(8) - 9(4) + 7(2) + 6 = 0$ $\quad \Rightarrow x - 2$ is a factor

$$\begin{array}{r|cccc} 2 & 2 & -9 & 7 & 6 \\ & & 4 & -10 & -6 \\ \hline & 2 & -5 & -3 & 0 \end{array}$$

$f(x) = (x-2)(2x^2 - 5x - 3)$

$f(x) = (x-2)(x-3)(2x+1)$

Zeros: $2, 3, -\frac{1}{2}$ } all w/ multiplicity 1

y-int: 6

(3) Degree: 5

Lead Coefficient: 1

$f(x) = x^5 - 9x^4 + 31x^3 - 51x^2 + 40x - 12$ $\quad \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

$P(1) = 1 - 9 + 31 - 51 + 40 - 12 = 0$

so $x-1$ is a factor

1	1	-9	31	-51	40	-12
		1	-8	23	-28	12
	1	-8	23	-28	12	0

$f(x) = (x-1)(x^4 - 8x^3 + 23x^2 - 28x + 12)$

$P(1) = 1 - 8 + 23 - 28 + 12 = 0$

so $x-1$ is a factor again

1	1	-8	23	-28	12
		1	-7	16	-12
	1	-7	16	-12	0

$f(x) = (x-1)^2 (x^3 - 7x^2 + 16x - 12)$

$P(1) = 1 - 7 + 16 - 12 = -2$

$P(-1) = -1 - 7 - 16 - 12 = -36$

$P(2) = 8 - 7(4) + 32 - 12 = 0$

so $x-2$ is a factor

2	1	-7	16	-12
		2	-10	12
	1	-5	6	0

$f(x) = (x-1)^2 (x-2)(x^2 - 5x + 6)$

$f(x) = (x-1)^2 (x-2)(x-2)(x-3)$

$f(x) = (x-1)^2 (x-2)^2 (x-3)$

③ continued

zeros: 1 w/ multiplicity 2
2 w/ multiplicity 2
3 w/ multiplicity 1

y-int: -12

④ Degree: 3
Lead Coefficient: 1

$f(x) = x^3 - 6x^2 + 11x - 6$ $\pm 1, \pm 2, \pm 3, \pm 6$

$P(1) = 1 - 6 + 11 - 6 = 0$ so $x-1$ is a factor

1	1	-6	11	-6
		1	-5	6
	1	-5	6	0

$f(x) = (x-1)(x^2 - 5x + 6)$
$f(x) = (x-1)(x-2)(x-3)$

zeros: 1, 2, 3 } all w/ multiplicity 1

y-int: -6

⑤ Degree: 3

Lead Coefficient: 1

$f(x) = x^3 + 6x^2 + 3x - 10$ $\quad \pm 1, \pm 2, \pm 5, \pm 10$

$P(1) = 1 + 6 + 3 - 10 = 0$ $\quad$ so $x-1$ is a factor

1	1	6	3	-10
		1	7	10
	1	7	10	0

$f(x) = (x-1)(x^2+7x+10)$

$f(x) = (x-1)(x+2)(x+5)$

Zeros: $1, -2, -5$ } all w/ multiplicity 1

y-int: -10

⑥ Degree: 4

Lead Coefficient: 1

$f(x) = x^4 + 0x^3 - 8x^2 + 0x + 16$ $\quad \pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

$P(1) = 1 - 8 + 16 = 9$

$P(-1) = 1 - 8 + 16 = 9$

$P(2) = 16 - 32 + 16 = 0$ so $x-2$ is a factor

2	1	0	-8	0	16
		2	4	-8	-16
	1	2	-4	-8	0

$f(x) = (x-2)(x^3 + 2x^2 - 4x - 8)$

$P(1) = 1 + 2 - 4 - 8 = -9$

$P(-1) = -1 + 2 + 4 - 8 = -3$

$P(2) = 8 + 8 - 8 - 8 = 0$

so $x-2$ is a factor again

2	1	2	-4	-8
		2	8	8
	1	4	4	0

$f(x) = (x-2)^2(x^2 + 4x + 4)$

$f(x) = (x-2)^2(x+2)(x+2)$

$f(x) = (x-2)^2(x+2)^2$

Zeros: 2 w/ multiplicity 2

-2 w/ multiplicity 2

y-int: 16